3

Hydraulic Turbines

3.1 INTRODUCTION

In a hydraulic turbine, water is used as the source of energy. Water or hydraulic turbines convert kinetic and potential energies of the water into mechanical power. The main types of turbines are (1) impulse and (2) reaction turbines. The predominant type of impulse machine is the Pelton wheel, which is suitable for a range of heads of about 150–2,000 m. The reaction turbine is further subdivided into the Francis type, which is characterized by a radial flow impeller, and the Kaplan or propeller type, which is an axial-flow machine. In the sections that follow, each type of hydraulic turbine will be studied separately in terms of the velocity triangles, efficiencies, reaction, and method of operation.

3.2 PELTON WHEEL

An American Engineer Lester A. Pelton discovered this (Fig. 3.1) turbine in 1880. It operates under very high heads (up to 1800 m.) and requires comparatively less quantity of water. It is a pure impulse turbine in which a jet of fluid delivered is by the nozzle at a high velocity on the buckets. These buckets are fixed on the periphery of a circular wheel (also known as runner), which is generally mounted on a horizontal shaft. The primary feature of the impulse

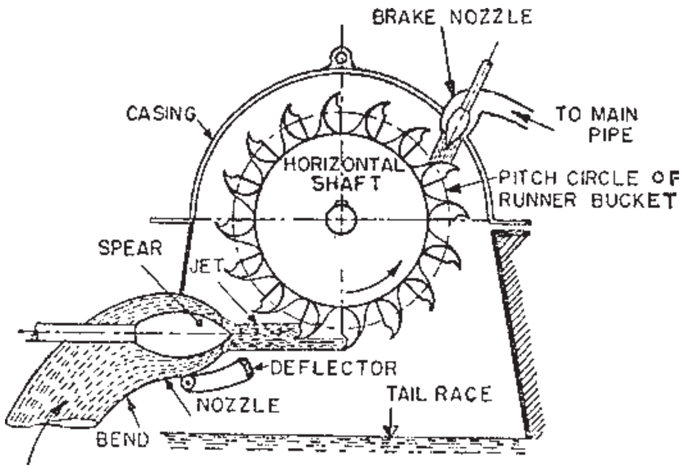

Figure 3.1 Single-jet, horizontal shaft Pelton turbine.

turbine with respect to fluid mechanics is the power production as the jet is deflected by the moving vane(s).

The impact of water on the buckets causes the runner to rotate and thus develops mechanical energy. The buckets deflect the jet through an angle of about 160 and 165° in the same plane as the jet. After doing work on the buckets water is discharged in the tailrace, and the whole energy transfer from nozzle outlet to tailrace takes place at constant pressure.

The buckets are so shaped that water enters tangentially in the middle and discharges backward and flows again tangentially in both the directions to avoid thrust on the wheel. The casing of a Pelton wheel does not perform any hydraulic function. But it is necessary to safeguard the runner against accident and also to prevent the splashing water and lead the water to the tailrace.

3.3 VELOCITY TRIANGLES

The velocity diagrams for the Pelton wheel are shown in Fig. 3.2.

Since the angle of entry of the jet is nearly zero, the inlet velocity triangle is a straight line, as shown in Fig. 3.2. If the bucket is brought to rest, then the relative fluid velocity, V_1, is given by

$$V_1 = \text{jet velocity} - \text{bucket speed}$$
$$= C_1 - U_1$$

The angle turned through by the jet in the horizontal plane during its passage over the bucket surface is α and the relative velocity at exit is V_2. The absolute

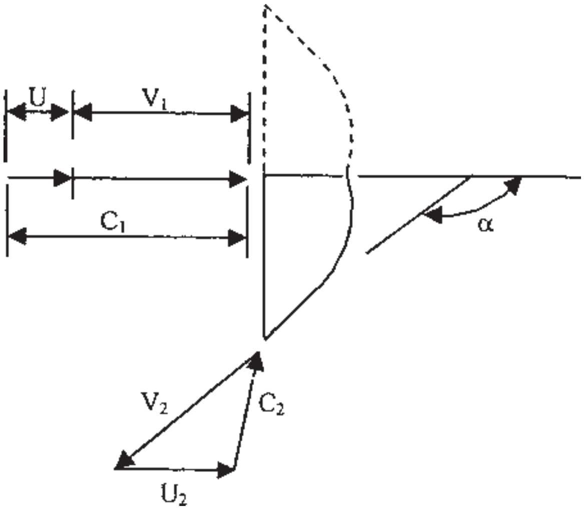

Figure 3.2 Velocity triangles for a Pelton wheel.

velocity, C_2, at exit can be obtained by adding bucket speed vector U_2 and relative velocity, V_2, at exit.

Now using Euler's turbine Eq. (1.78)

$$W = U_1 C_{W1} - U_2 C_{W2}$$

Since in this case C_{W2} is in the negative x direction,

$$W = U\{(U + V_1) + [V_1 \cos(180 - \alpha) - U]\}$$

Neglecting loss due to friction across the bucket surface, that is, $V_1 = V_2$, then

$$W = U(V_1 - V_1 \cos \alpha)$$

Therefore

$$E = U(C_1 - U)(1 - \cos \alpha)/g \tag{3.1}$$

the units of E being Watts per Newton per second weight of flow.

Eq. (3.1) can be optimized by differentiating with respect to U, and equating it to zero.

Therefore

$$\frac{dE}{dU} = (1 - \cos \alpha)(C_1 - 2U)/g = 0$$

Then

$$C_1 = 2U \text{ or } \quad U = C_1/2 \tag{3.2}$$

Substituting Eq. (3.2) into Eq. (3.1) we get

$$E_{\text{max}} = C_1^2(1 - \cos\alpha)/4g$$

In practice, surface friction is always present and $V_1 \neq V_2$, then Eq. (3.1) becomes

$$E = U(C_1 - U)(1 - k\cos\alpha)/g \tag{3.3}$$

where $k = \frac{V_2}{V_1}$

Introducing hydraulic efficiency as

$$\eta_{\text{h}} = \frac{\text{Energy Transferred}}{\text{Energy Available in jet}}$$

$$\text{i.e.}\quad \eta_{\text{h}} = \frac{E}{(C_1^2/2g)} \tag{3.4}$$

if $\alpha = 180°$, the maximum hydraulic efficiency is 100%. In practice, deflection angle is in the order of 160–165°.

3.4 PELTON WHEEL (LOSSES AND EFFICIENCIES)

Head losses occur in the pipelines conveying the water to the nozzle due to friction and bend. Losses also occur in the nozzle and are expressed by the velocity coefficient, C_{v}.

The jet efficiency (η_{j}) takes care of losses in the nozzle and the mechanical efficiency (η_{m}) is meant for the bearing friction and windage losses. The overall efficiency (η_{o}) for large Pelton turbine is about 85–90%. Following efficiency is usually used for Pelton wheel.

$$\text{Pipeline transmission efficiency} = \frac{\text{Energy at end of the pipe}}{\text{Energy available at reservoir}}$$

Figure 3.3 shows the total headline, where the water supply is from a reservoir at a head H_1 above the nozzle. The frictional head loss, h_{f}, is the loss as the water flows through the pressure tunnel and penstock up to entry to the nozzle.

Then the transmission efficiency is

$$\eta_{\text{trans}} = (H_1 - h_{\text{f}})/H_1 = H/H_1 \tag{3.5}$$

The nozzle efficiency or jet efficiency is

$$\eta_{\text{j}} = \frac{\text{Energy at nozzle outlet}}{\text{Energy at nozzle inlet}} = C_1^2/2gH \tag{3.6}$$

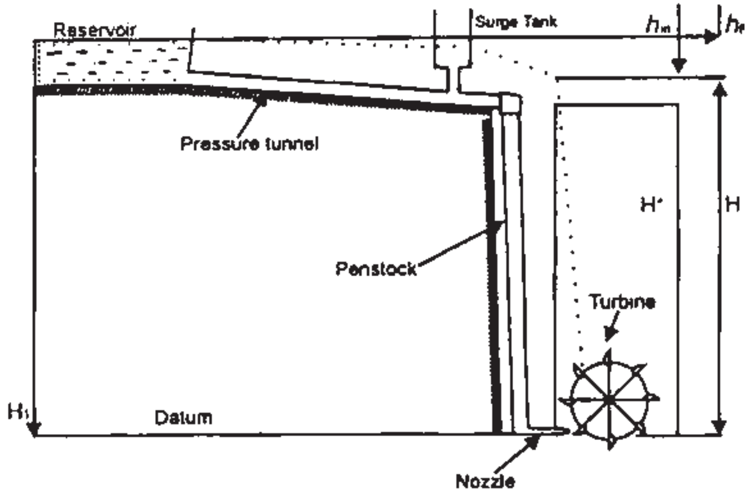

Figure 3.3 Schematic layout of hydro plant.

Nozzle velocity coefficient

$$C_v = \frac{\text{Actual jet velocity}}{\text{Theoretical jet velocity}} = C_1 / \sqrt{2gH}$$

Therefore the nozzle efficiency becomes

$$\eta_j = C_1^2 / 2gH = C_v^2 \tag{3.7}$$

The characteristics of an impulse turbine are shown in Fig. 3.4.

Figure 3.4 shows the curves for constant head and indicates that the peak efficiency occurs at about the same speed ratio for any gate opening and that

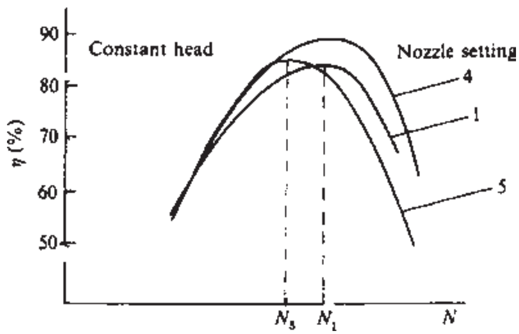

Figure 3.4 Efficiency vs. speed at various nozzle settings.

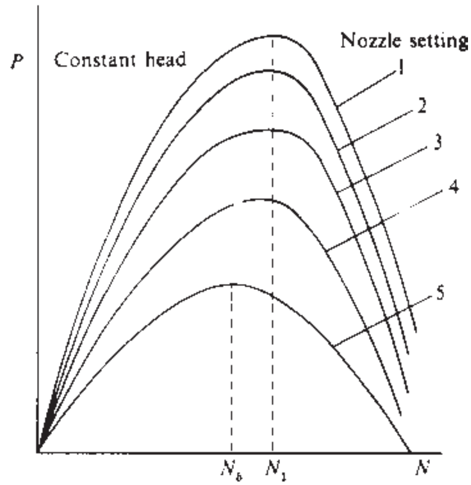

Figure 3.5 Power vs. speed of various nozzle setting.

the peak values of efficiency do not vary much. This happens as the nozzle velocity remaining constant in magnitude and direction as the flow rate changes, gives an optimum value of U/C_1 at a fixed speed. Due to losses, such as windage, mechanical, and friction cause the small variation. Fig. 3.5 shows the curves for power vs. speed. Fixed speed condition is important because generators are usually run at constant speed.

Illustrative Example 3.1: A generator is to be driven by a Pelton wheel with a head of 220 m and discharge rate of 145 L/s. The mean peripheral velocity of wheel is 14 m/s. If the outlet tip angle of the bucket is 160°, find out the power developed.

Solution:

$$\text{Dischargerate}, Q = 145\,\text{L/s}$$

$$\text{Head}, H = 220\,\text{m}$$

$$U_1 = U_2 = 14\,\text{m/s}$$

$$\beta_2 = 180 - 160° = 20°$$

Refer to Fig. 3.6

Using Euler's equation, work done per weight mass of water per sec.

$$= (C_{w1}U_1 - C_{w2}U_2)$$

But for Pelton wheel C_{w2} is negative

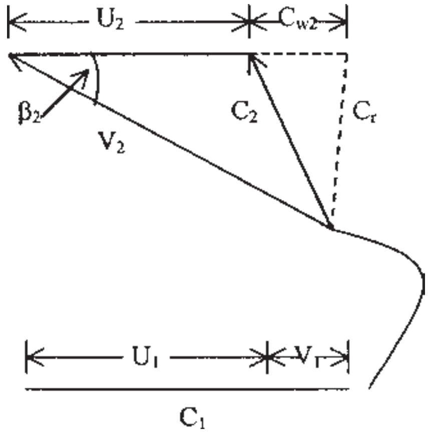

Figure 3.6 Inlet and outlet velocity triangles.

Therefore

$$\text{Work done / s} = (C_{w1}U_1 + C_{w2}U_2)\ \text{Nm / s}$$

From inlet velocity triangle

$$C_{w1} = C_1 \text{ and } \frac{C_1^2}{2g} = H$$

Hence, $C_1 = \sqrt{2gH} = \sqrt{2 \times 9.81 \times 220} = 65.7$ m/s
Relative velocity at inlet is

$$V_1 = C_1 - U_1 = 65.7 - 14 = 51.7 \text{ m/s}$$

From outlet velocity triangle

$$V_1 = V_2 = 51.7 \text{ m/s(neglecting friction)}$$

and cos $\beta_2 = \frac{U_2 + C_{w2}}{V_2}$ or

$$\cos(20) = \frac{14 + C_{w2}}{51.7}$$

Therefore

$$C_{w2} = 34.58 \text{ m/s}$$

Hence, work done per unit mass of water per sec.

$$= (65.7)(14) + (34.58)(14) = 1403.92 \text{ Nm}$$

$$\text{Power developed} = \frac{(1403.92)(145)}{1000} = 203.57 \text{ kW}$$

Design Example 3.2: A Pelton wheel is supplied with 0.035 m^3/s of water under a head of 92 m. The wheel rotates at 725 rpm and the velocity coefficient of the nozzle is 0.95. The efficiency of the wheel is 82% and the ratio of bucket speed to jet speed is 0.45. Determine the following:

1. Speed of the wheel
2. Wheel to jet diameter ratio
3. Dimensionless power specific speed of the wheel

Solution:

Overall efficiency $\eta_o = \dfrac{\text{Power developed}}{\text{Power available}}$

$$\therefore\ P = \rho g Q H \eta_o \text{ J/s} = \frac{\rho g Q H \eta_o}{1000} \text{ kW}$$
$$= 9.81(0.035)(92)(0.82) = 25.9 \text{ kW}$$

Velocity coefficient

$$C_v = \frac{C_1}{\sqrt{2gH}}$$

or $C_1 = C_v\sqrt{2gH} = 0.95[(2)(9.81)(92)]^{1/2} = 40.36$ m/s

1. Speed of the wheel is given by

$$U = 0.45(40.36) = 18.16 \text{ m/s}$$

2. If D is the wheel diameter, then

$$U = \frac{\omega D}{2} \quad or \quad D = \frac{2U}{\omega} = \frac{(2)(18.16)(60)}{725(2\pi)} = 0.478 \text{ m}$$

Jet area $A = \dfrac{Q}{C_1} = \dfrac{0.035}{40.36} = 0.867 \times 10^{-3} \text{m}^2$

and Jet diameter, d, is given by

$$d = \left(\frac{4A}{\pi}\right)^{1/2} = \left(\frac{(4)(0.867 \times 10^{-3})}{\pi}\right)^{1/2} = 0.033 \text{ m}$$

Diameter ratio $\dfrac{D}{d} = \dfrac{0.478}{0.033} = 14.48$

3. Dimensionless specific speed is given by Eq. (1.10)

$$
\begin{aligned}
N\text{sp} &= \frac{NP^{1/2}}{\rho^{1/2}(gH)^{5/4}} \\
&= \left(\frac{725}{60}\right) \times \left(\frac{(25.9)(1000)}{10^3}\right)^{1/2} \times \left(\frac{1}{(9.81)\times(92)}\right)^{5/4} \\
&= (12.08)(5.09)(0.0002) \\
&= 0.0123 \text{ rev} \\
&= (0.0123)(2\pi) \text{ rad} \\
&= 0.077 \text{ rad}
\end{aligned}
$$

Illustrative Example 3.3: The speed of Pelton turbine is 14 m/s. The water is supplied at the rate of 820 L/s against a head of 45 m. If the jet is deflected by the buckets at an angle of 160°, find the hP and the efficiency of the turbine.

Solution:
Refer to Fig. 3.7

$U_1 = U_2 = 14$ m/s

$Q = 820$ L/s $= 0.82$ m^3/s

$H = 45$ m

$\beta_2 = 180 - 160° = 20°$

Velocity of jet

$$
\begin{aligned}
C_1 &= C_V\sqrt{2gH}, \text{ assuming } C_V = 0.98 \\
&= 0.98\sqrt{(2)(9.81)(45)} = 29.12 \text{ m/s}
\end{aligned}
$$

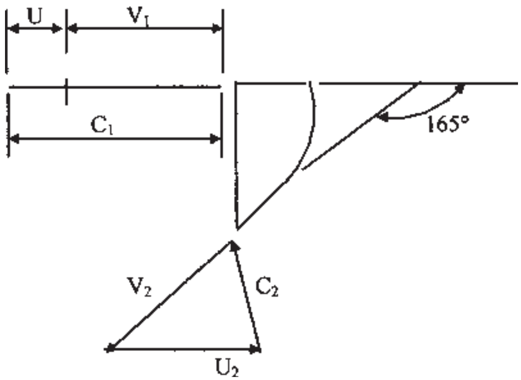

Figure 3.7 Velocity triangle for Example 3.3.

Assuming

$$\beta_1 = 180^\circ$$
$$\beta_2 = 180 - 160^\circ = 20^\circ$$
$$C_{w1} = C_1 = 29.12\,\text{m/s}$$
$$V_1 = C_1 - U_1 = 29.12 - 14 = 15.12\,\text{m/s}$$

From outlet velocity triangle,

$$U_1 = U_2 (\text{neglecting losses on buckets})$$

$$V_2 = 15.12\,\text{m/s} \quad \text{and} \quad U_2 = 14\,\text{m/s}$$

$$C_{w2} = V_2 \cos\alpha_2 - U_2 = 15.12 \cos 20^\circ - 14$$
$$= 0.208\,\text{m/s}$$

Work done per weight mass of water per sec

$$= (C_{w1} + C_{w2})U$$
$$= (29.12 + 0.208) \times (14) = 410.6\,\text{Nm/s}$$

$$\therefore \text{ Power developed} = \frac{(410.6)(0.82 \times 10^3)}{1000} = 336.7\,\text{kW}$$
$$= 451\,\text{hP}$$

$$\text{Efficiency}\,\eta_1 = \frac{\text{Power developed}}{\text{Available Power}}$$
$$= \frac{(1000)(336.7)}{(1000)(9.81)(0.82)(45)} = 0.930 \text{ or } 93.0\%$$

Illustrative Example 3.4: A Pelton wheel develops 12,900 kW at 425 rpm under a head of 505 m. The efficiency of the machine is 84%. Find (1) discharge of the turbine, (2) diameter of the wheel, and (3) diameter of the nozzle. Assume $C_v = 0.98$, and ratio of bucket speed to jet speed $= 0.46$.

Solution:

Head, $H = 505$ m.

Power, $P = 12{,}900$ kW

Speed, $N = 425$ rpm

Efficiency, $\eta_o = 84\%$

1. Let Q be the discharge of the turbine

$$\text{Using the relation } \eta_o = \frac{P}{9.81QH}$$

or

$$0.84 = \frac{12,900}{(9.81)(505)Q} = \frac{2.60}{Q}$$

or

$$Q = 3.1\ \text{m}^3/\text{s}$$

2. Velocity of jet

$$C = C_V\sqrt{2gH}\ (\text{assume}\ \ C_V = 0.98)$$

or

$$C = 0.98\sqrt{(2)(9.81)(505)} = 97.55\ \text{m/s}$$

Tangential velocity of the wheel is given by

$$U = 0.46C = (0.46)(97.55) = 44.87\ \text{m/s}$$

and

$$U = \frac{\pi DN}{60},\ \text{hence wheel diameter is}$$

$$D = \frac{60U}{\pi N} = \frac{(60)(44.87)}{(\pi)(425)} = 2.016\ \text{m}$$

3. Let d be the diameter of the nozzle
The discharge through the nozzle must be equal to the discharge of the turbine. Therefore

$$Q = \tfrac{\pi}{4} \times d^2 \times C$$

$$3.1 = (\tfrac{\pi}{4})(d^2)(97.55) = 76.65\, d^2$$

$$\therefore\ d = \sqrt{\frac{3.1}{76.65}} = 0.20\ \text{m}$$

Illustrative Example 3.5: A double Overhung Pelton wheel unit is to operate at 12,000 kW generator. Find the power developed by each runner if the generator is 95%.

Solution:
Output power = 12,000 kW

Efficiency, $\eta = 95\%$

Therefore, power generated by the runner

$$= \frac{12,000}{0.95} = 12,632\ \text{kW}$$

Since there are two runners, power developed by each runner

$$= \frac{12,632}{2} = 6316\,\text{kW}$$

Design Example 3.6: At the power station, a Pelton wheel produces 1260 kW under a head of 610 m. The loss of head due to pipe friction between the reservoir and nozzle is 46 m. The buckets of the Pelton wheel deflect the jet through an angle of 165°, while relative velocity of the water is reduced by 10% due to bucket friction. The bucket/jet speed ratio is 0.46. The bucket circle diameter of the wheel is 890 mm and there are two jets. Find the theoretical hydraulic efficiency, speed of rotation of the wheel, and diameter of the nozzle if the actual hydraulic efficiency is 0.9 times that calculated above. Assume nozzle velocity coefficient, $C_v = 0.98$.

Solution:
Refer to Fig. 3.8.

$$\text{Hydraulic efficiency } \eta_h = \frac{\text{Power output}}{\text{Energy available in the jet}} = \frac{P}{0.5mC_1^2}$$

At entry to nozzle

$$H = 610 - 46 = 564\,\text{m}$$

Using nozzle velocity coefficient

$$C_1 = C_V\sqrt{2gH} = 0.98\sqrt{(2)(9.81)(564)} = 103.1\,\text{m/s}$$

Now

$$\frac{W}{m} = U_1C_{w1} - U_2C_{w2}$$
$$= U\{(U + V_1) - [U - V_2\cos(180° - \alpha)]\}$$
$$= U[(C_1 - U)(1 - k\cos\alpha)] \text{ where } V_2 = kV_1$$

Therefore, $W/m = 0.46C_1(C_1 - 0.46C_1)(1 - 0.9\cos 165°)$
Substitute the value of C_1

$$W/m = 5180.95$$

$$\text{Theoretical hydraulic efficiency} = \frac{\text{Power output}}{\text{Energy available in the jet}}$$
$$= \frac{5180.95}{0.5 \times 103^2} = 98\%$$

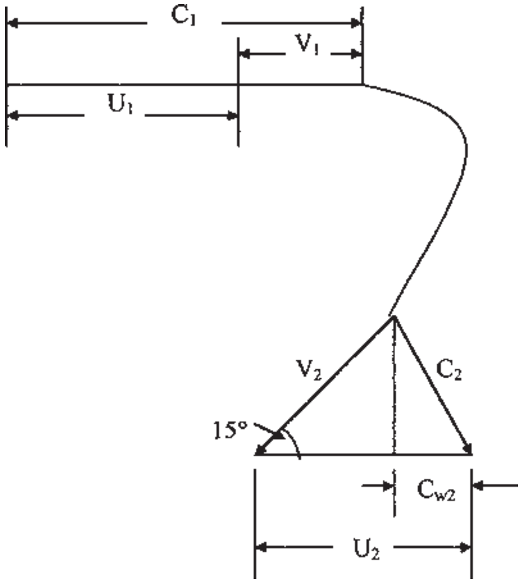

Figure 3.8 Velocity triangle for Example 3.6.

Actual hydraulic efficiency = (0.9)(0.98) = 0.882

Wheel bucket speed = (0.46)(103) = 47.38 m/s

$$\text{Wheel rotational speed} = N = \frac{(47.38)(60)}{(0.445)(2\pi)} = 1016\,\text{rpm}$$

$$\text{Actual hydraulic efficiency} = \frac{\text{Actual power}}{\text{energy in the jet}} = \frac{(1260 \times 10^3)}{0.5\,mC_1^2}$$

$$\text{Therefore, } m = \frac{(1260 \times 10^3)}{(0.882)(0.5)(103^2)} = 269\,\text{kg/s}$$

For one nozzle, $m = 134.5$ kg/s

For nozzle diameter, using continuity equation, $m = \rho C_1 A = \dfrac{\rho C_1 \pi d^2}{4}$

$$\text{Hence, } d = \sqrt{\frac{(134.5)(4)}{(\pi)(103 \times 10^3)}} = 0.041\,\text{m} = 41\,\text{mm}$$

Illustrative Example 3.7: A Pelton wheel has a head of 90 m and head lost due to friction in the penstock is 30 m. The main bucket speed is 12 m/s and the nozzle discharge is 1.0 m^3/s. If the bucket has an angle of 15° at the outlet and $C_v = 0.98$, find the power of Pelton wheel and hydraulic efficiency.

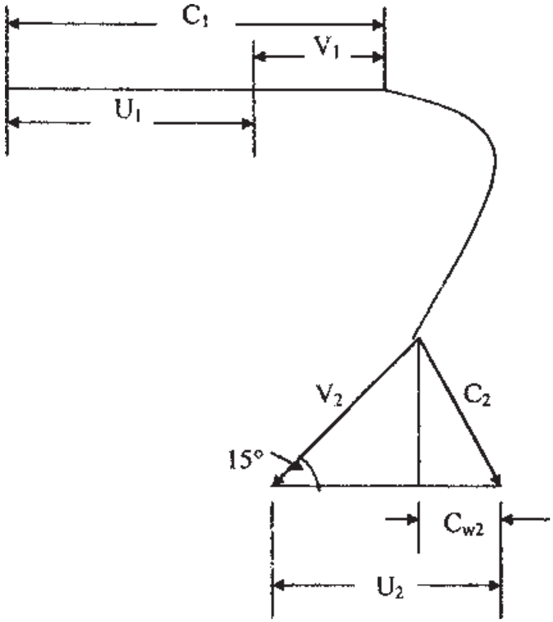

Figure 3.9 Velocity triangle for Example 3.7.

Solution: (Fig. 3.9)
Head = 90 m
Head lost due to friction = 30 m
Head available at the nozzle = 90 − 30 = 60 m
$Q = 1\,\text{m}^3/\text{s}$
From inlet diagram

$$C_1 = C_V\sqrt{2gH} = 0.98 \times \sqrt{(2)(9.81)(60)} = 33.62\,\text{m/s}$$

Therefore, $V_1 = C_1 - U_1 = 33.62 - 12 = 21.62\,\text{m/s}$
From outlet velocity triangle

$$V_2 = V_1 = 21.16\,\text{m/s (neglecting losses)}$$

$$U_2 = U_1 = 12\,\text{m/s}$$

$$C_{w2} = V_2 \cos\alpha - U_2 = 21.62\cos 15^\circ - 12 = 8.88\,\text{m/s}$$

and

$$Cr_2 = V_2 \sin\alpha = 21.62 \sin 15° = 5.6\,\text{m/s}$$

Therefore,

$$C_2 = \sqrt{C_{w2}^2 + Cr_2^2} = \sqrt{(8.88)^2 + (5.6)^2} = 10.5\,\text{m/s}$$

$$\therefore \text{ Work done} = \frac{C_1^2 - C_2^2}{2} = \frac{(33.62)^2 - (10.5)^2}{2} = 510\,\text{kJ/kg}$$

Note Work done can also be found by using Euler's equation ($C_{w1}U_1 + C_{w2}U_2$)

Power = 510 kW

Hydraulic efficiency

$$\eta_h = \frac{\text{work done}}{\text{kinetic energy}} = \frac{(510)(2)}{(33.62)^2} = 90.24\%$$

Design Example 3.8: A single jet Pelton wheel turbine runs at 305 rpm against a head of 515 m. The jet diameter is 200 mm, its deflection inside the bucket is 165° and its relative velocity is reduced by 12% due to friction. Find (1) the waterpower, (2) resultant force on the bucket, (3) shaft power if the mechanical losses are 4% of power supplied, and (4) overall efficiency. Assume necessary data.

Solution: (Fig. 3.10)
Velocity of jet, $C_1 = C_V\sqrt{2gH} = 0.98\sqrt{(2)(9.81)(515)} = 98.5\,\text{m/s}$
Discharge, Q is given by

$$Q = \text{Area of jet} \times \text{Velocity} = \frac{\pi}{4} \times (0.2)^2(98.5) = 3.096\,\text{m}^3/\text{s}$$

1. Water power is given by

$$P = \rho g Q H = (9.81)(3.096)(515) = 15641.5\,\text{kW}$$

2. Bucket velocity, U_1, is given by

$$U_1 = C_V\sqrt{2gH}$$
$$= 0.46\sqrt{(2)(9.81)(515)} = 46\,\text{m/s (assuming } C_V = 0.46)$$

Relative velocity, V_1, at inlet is given by

$$V_1 = C_1 - U_1 = 98.5 - 46 = 52.5\,\text{m/s}$$

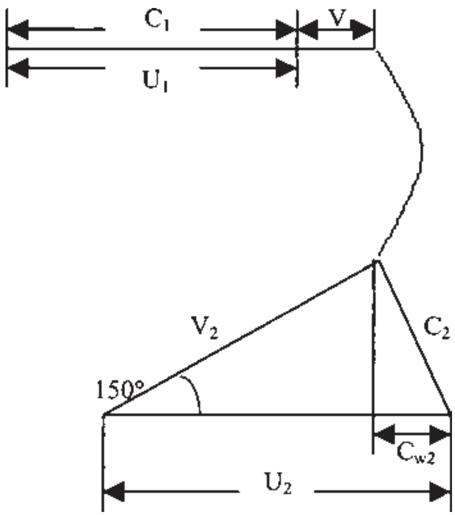

Figure 3.10 Velocity triangles for Example 3.8.

and

$$V_2 = 0.88 \times 52.5 = 46.2\,\text{m/s}$$

From the velocity diagram

$$C_{w2} = U_2 - V_2 \cos 15 = 46 - 46.2 \times 0.966 = 1.37\,\text{m/s}$$

Therefore force on the bucket

$$= \rho Q(C_{w1} - C_{w2}) = 1000 \times 3.096(98.5 - 1.37)$$
$$= 300714\,\text{N}$$

3. Power produced by the Pelton wheel

$$= \frac{(300714)(46)}{1000} = 13832.8\,\text{kW}$$

Taking mechanical loss = 4%

Therefore, shaft power produced = 0.96 × 13832.8 = 13279.5 kW

4. Overall efficiency

$$\eta_o = \frac{13279.5}{15641.5} = 0.849 \text{ or } 84.9\%$$

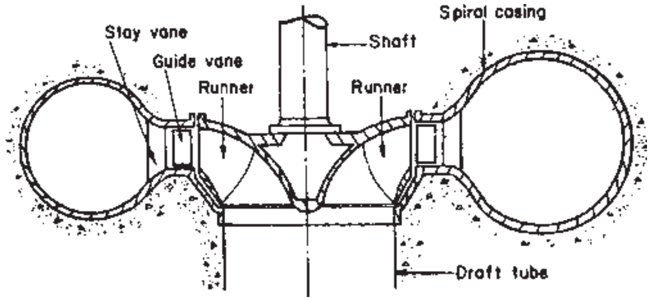

Figure 3.11 Outlines of a Francis turbine.

3.5 REACTION TURBINE

The radial flow or Francis turbine is a reaction machine. In a reaction turbine, the runner is enclosed in a casing and therefore, the water is always at a pressure other than atmosphere. As the water flows over the curved blades, the pressure head is transformed into velocity head. Thus, water leaving the blade has a large relative velocity but small absolute velocity. Therefore, most of the initial energy of water is given to the runner. In reaction turbines, water leaves the runner at atmospheric pressure. The pressure difference between entrance and exit points of the runner is known as reaction pressure.

The essential difference between the reaction rotor and impulse rotor is that in the former, the water, under a high station head, has its pressure energy converted into kinetic energy in a nozzle. Therefore, part of the work done by the fluid on the rotor is due to reaction from the pressure drop, and part is due to a change in kinetic energy, which represents an impulse function. Fig. 3.11 shows a cross-section through a Francis turbine and Fig. 3.12 shows an energy distribution through a hydraulic reaction turbine. In reaction turbine, water from the reservoir enters the turbine casing through penstocks.

Hence, the total head is equal to pressure head plus velocity head. Thus, the water enters the runner or passes through the stationary vanes, which are fixed around the periphery of runners. The water then passes immediately into the rotor where it moves radially through the rotor vanes and exits from the rotor blades at a smaller diameter, after which it turns through 90° into the draft tube. The draft tube is a gradually increasing cross-sectional area passage. It helps in increasing the work done by the turbine by reducing pressure at the exit. The penstock is a waterway, which carries water from the reservoir to the turbine casing. The inlet and outlet velocity triangles for the runner are shown in Fig. 3.13.

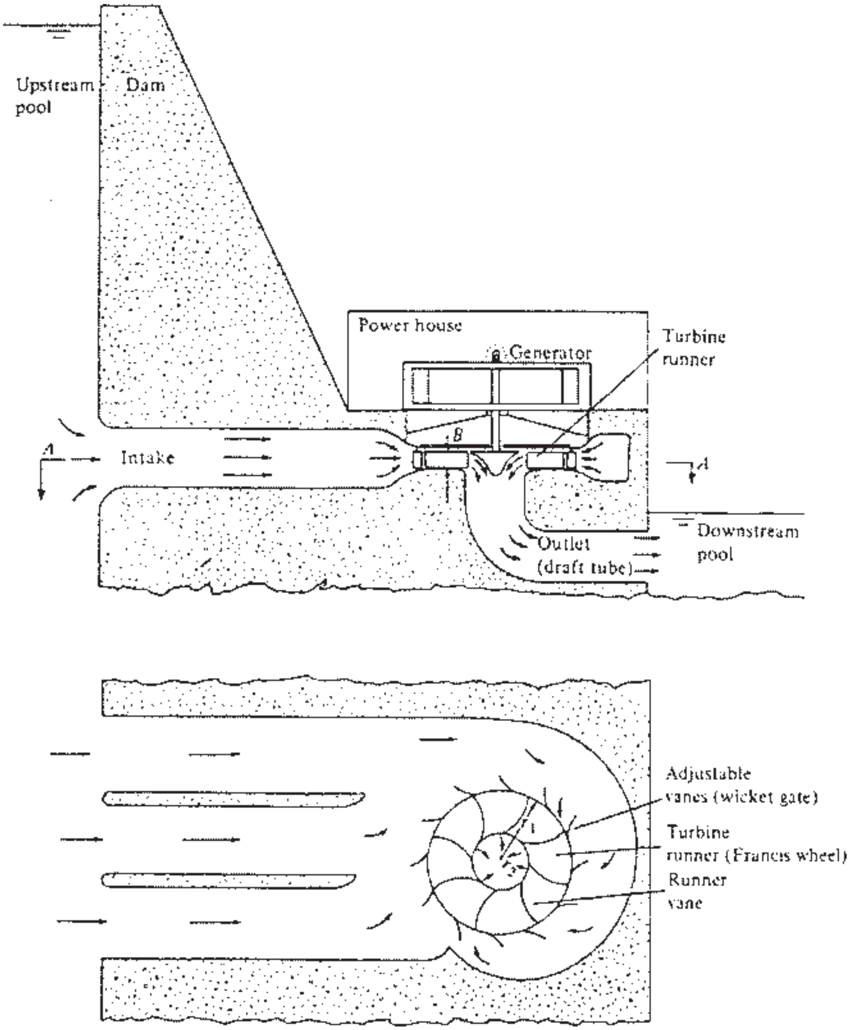

Figure 3.12 Reaction turbine installation.

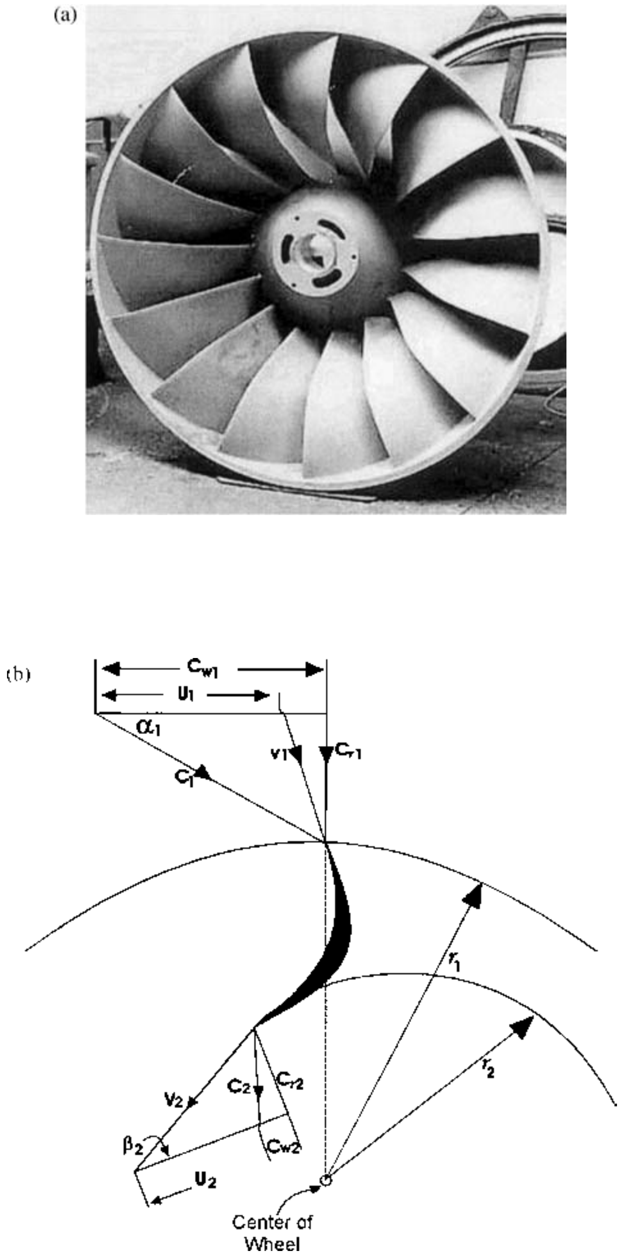

Figure 3.13 (a) Francis turbine runner and (b) velocity triangles for inward flow reaction turbine.

Let

C_1 = Absolute velocity of water at inlet

D_1 = Outer diameter of the runner

N = Revolution of the wheel per minute

U_1 = Tangential velocity of wheel at inlet

V_1 = Relative velocity at inlet

C_{r1} = radial velocity at inlet

α_1 = Angle with absolute velocity to the direction of motion

β_1 = Angle with relative velocity to the direction of motion

H = Total head of water under which turbine is working

$C_2, D_2, U_2, V_2, C_{r2}$ = Corresponding values at outlet

Euler's turbine equation Eq. (1.78) and E is maximum when C_{w2} (whirl velocity at outlet) is zero that is when the absolute and flow velocities are equal at the outlet.

3.6 TURBINE LOSSES

Let

P_s = Shaft power output

P_m = Mechanical power loss

P_r = Runner power loss

P_c = Casing and draft tube loss

P_l = Leakage loss

P = Water power available

$P_h = P_r + P_c + P_l$ = Hydraulic power loss

Runner power loss is due to friction, shock at impeller entry, and flow separation. If h_f is the head loss associated with a flow rate through the runner of Q_r, then

$$P_S = \rho g Q_r h_f \text{ (Nm/s)} \tag{3.8}$$

Leakage power loss is due to leakage in flow rate, q, past the runner and therefore not being handled by the runner. Thus

$$Q = Q_r + q \tag{3.9}$$

If H_r is the head across the runner, the leakage power loss becomes

$$P_l = \rho g H_r q \,(\mathrm{Nm/s}) \tag{3.10}$$

Casing power loss, P_c, is due to friction, eddy, and flow separation losses in the casing and draft tube. If h_c is the head loss in casing then

$$P_c = \rho g Q h_c \,(\mathrm{Nm/s}) \tag{3.11}$$

From total energy balance we have

$$\rho g Q H = P_m + \rho g (h_f Q_r + h_c Q + H_r q + P_s)$$

Then overall efficiency, η_o, is given by

$$\eta_o = \frac{\text{Shaft power output}}{\text{Fluid power available at inlet}}$$

or

$$\eta_o = \frac{P_s}{\rho g Q H} \tag{3.12}$$

Hydraulic efficiency, η_h, is given by

$$\eta_h = \frac{\text{Power available at runner}}{\text{Fluid power available at inlet}}$$

or

$$\eta_h = \frac{(P_s + P_m)}{\rho g Q H} \tag{3.13}$$

Eq. (3.13) is the theoretical energy transfer per unit weight of fluid. Therefore the maximum efficiency is

$$\eta_h = U_1 C_{w1}/gH \tag{3.14}$$

3.7 TURBINE CHARACTERISTICS

Part and overload characteristics of Francis turbines for specific speeds of 225 and 360 rpm are shown in Fig. 3.14

Figure 3.14 shows that machines of low specific speeds have a slightly higher efficiency. It has been experienced that the Francis turbine has unstable characteristics for gate openings between 30 to 60%, causing pulsations in output and pressure surge in penstocks. Both these problems were solved by Paul Deriaz by designing a runner similar to Francis runner but with adjustable blades.

The part-load performance of the various types are compared in Fig. 3.15 showing that the Kaplan and Pelton types are best adopted for a wide range of load but are followed fairly closely by Francis turbines of low specific speed.

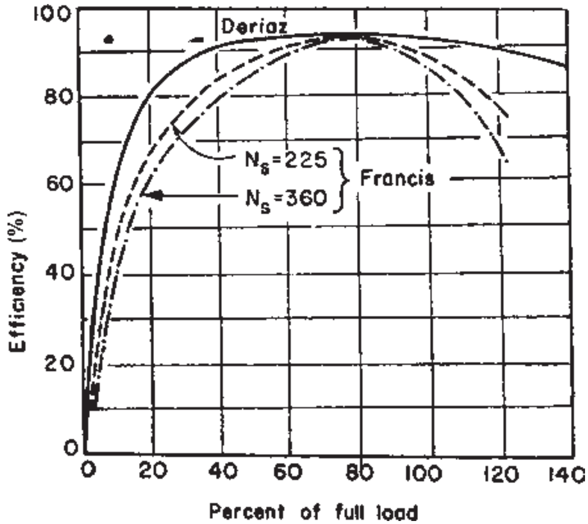

Figure 3.14 Variation of efficiency with load for Francis turbines.

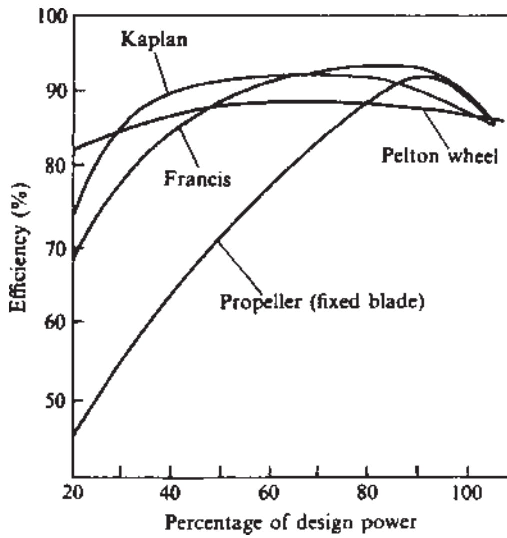

Figure 3.15 Comparison of part-load efficiencies of various types of hydraulic turbine.

3.8 AXIAL FLOW TURBINE

In an axial flow reaction turbine, also known as Kaplan turbine, the flow of water is parallel to the shaft.

A Kaplan turbine is used where a large quantity of water is available at low heads and hence the blades must be long and have large chords so that they are strong enough to transmit the very high torque that arises. Fig. 3.16 and 3.17 shows the outlines of the Kaplan turbine. The water from the scroll flows over the guide blades and then over the vanes. The inlet guide vanes are fixed and are situated at a plane higher than the runner blades such that fluid must turn through 90° to enter the runner in the axial direction. The function of the guide vanes is to impart whirl to the fluid so that the radial distribution of velocity is the same as in a free vortex.

Fig. 3.18 shows the velocity triangles and are usually drawn at the mean radius, since conditions change from hub to tip. The flow velocity is axial at inlet and outlet, hence $C_{r1} = C_{r2} = C_a$

C_1 is the absolute velocity vector at angle α_1 to U_1, and V_1 is the relative velocity at an angle β_1. For maximum efficiency, the whirl component $C_{w2} = 0$, in which case the absolute velocity at exit is axial and then $C_2 = C_{r2}$

Using Euler's equation

$$E = U(C_{w1} - C_{w2})/g$$

and for zero whirl ($C_{w2} = 0$) at exit

$$E = UC_{w1}/g$$

3.9 CAVITATION

In the design of hydraulic turbine, cavitation is an important factor. As the outlet velocity V_2 increases, then p_2 decreases and has its lowest value when the vapor pressure is reached.

At this pressure, cavitation begins. The Thoma parameter $\sigma = \frac{\text{NPSH}}{H}$ and Fig. 3.19 give the permissible value of σ_c in terms of specific speed.

The turbines of high specific speed have a high critical value of σ, and must therefore be set lower than those of smaller specific speed (N_s).

Illustrative Example 3.9: Consider an inward flow reaction turbine in which velocity of flow at inlet is 3.8 m/s. The 1 m diameter wheel rotates at 240 rpm and absolute velocity makes an angle of 16° with wheel tangent. Determine (1) velocity of whirl at inlet, (2) absolute velocity of water at inlet, (3) vane angle at inlet, and (4) relative velocity of water at entrance.

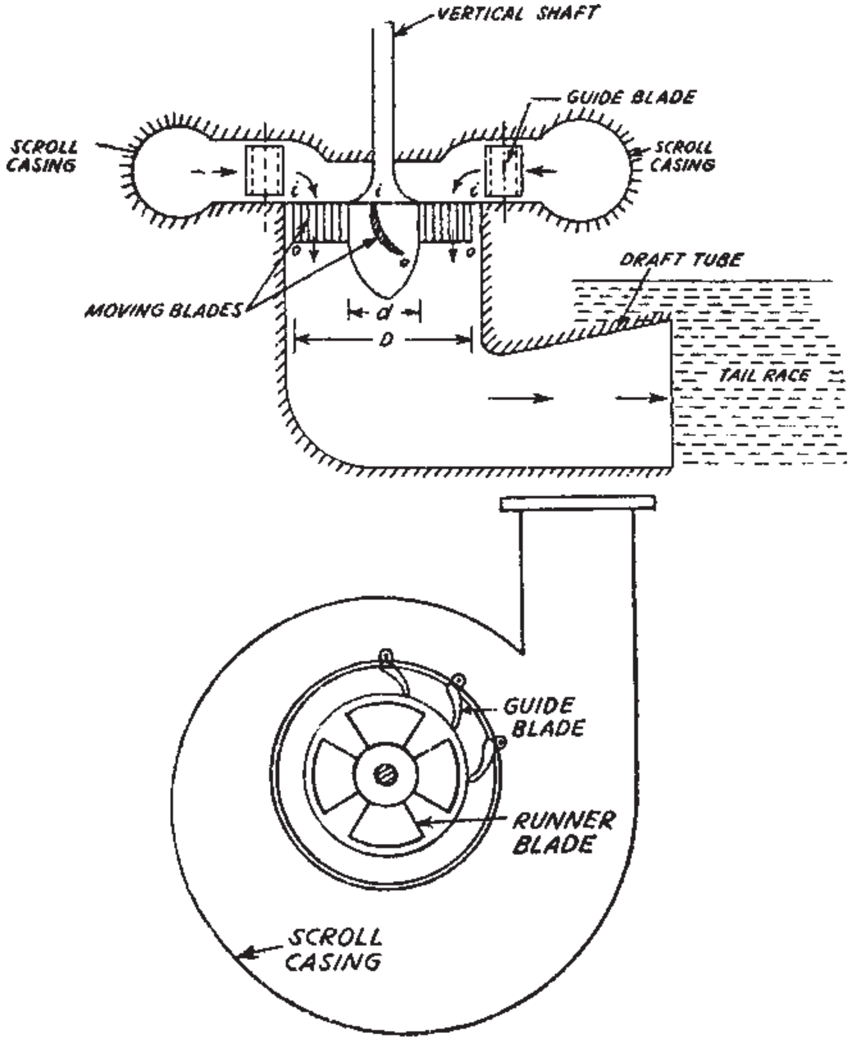

Figure 3.16 Kaplan turbine of water is available at low heads.

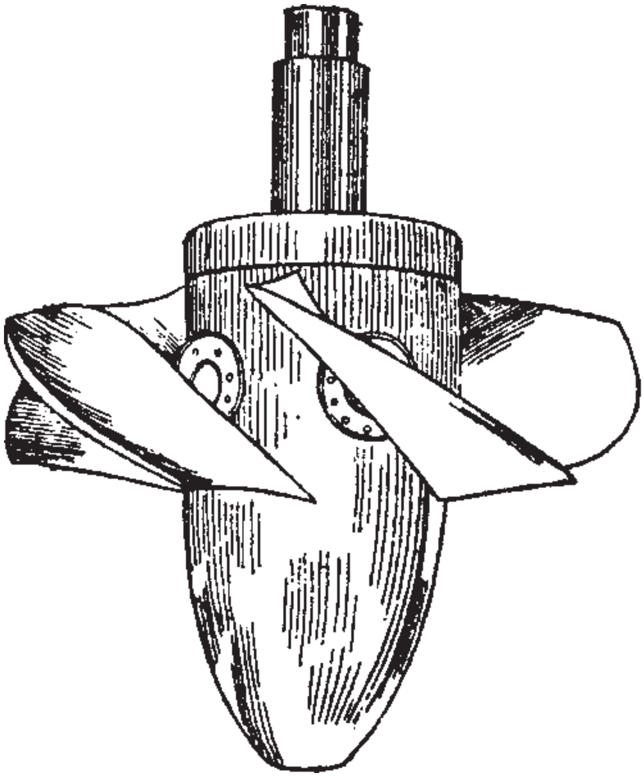

Figure 3.17 Kaplan turbine runner.

Solution: From Fig. 3.13b

1. From inlet velocity triangle (subscript 1)

$$\tan\alpha_1 = \frac{C_{r1}}{C_{w1}} \quad \text{or} \quad C_{w1} = \frac{C_{r1}}{\tan\alpha_1} = \frac{3.8}{\tan 16^\circ} = 13.3\,\text{m/s}$$

2. Absolute velocity of water at inlet, C_1, is

$$\sin\alpha_1 = \frac{C_{r1}}{C_1} \quad \textit{or} \quad C_1 = \frac{C_{r1}}{\sin\alpha_1} = \frac{3.8}{\sin 16^\circ} = 13.79\,\text{m/s}$$

3.

$$U_1 = \frac{(\pi D_1)(N)}{60} = \frac{(\pi)(1)(240)}{60} = 12.57\,\text{m/s}$$

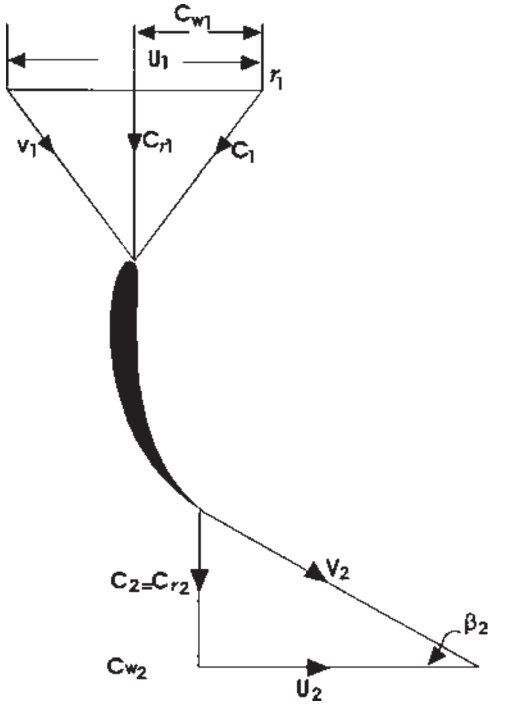

Figure 3.18 Velocity triangles for an axial flow hydraulic turbine.

and

$$\tan \beta_1 = \frac{C_{r1}}{(C_{w1} - U_1)} = \frac{3.8}{(13.3 - 12.57)} = \frac{3.8}{0.73} = 5.21$$

$$\therefore \ \beta_1 = 79^\circ \text{ nearby}$$

4. Relative velocity of water at entrance

$$\sin \beta_1 = \frac{C_{r1}}{V_1} \quad or \quad V_1 = \frac{C_{r1}}{\sin \beta_1} = \frac{3.8}{\sin 79^\circ} = 3.87 \text{m/s}$$

Illustrative Example 3.10: The runner of an axial flow turbine has mean diameter of 1.5 m, and works under the head of 35 m. The guide blades make an angle of 30° with direction of motion and outlet blade angle is 22°. Assuming axial discharge, calculate the speed and hydraulic efficiency of the turbine.

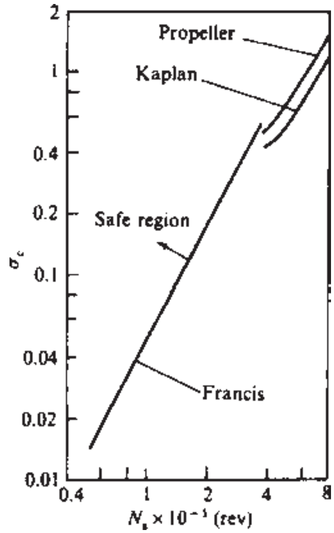

Figure 3.19 Cavitation limits for reaction turbines.

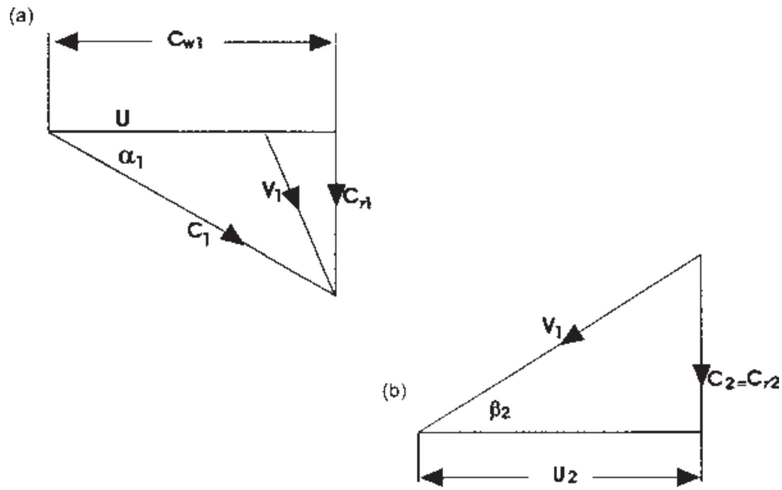

Figure 3.20 Velocity triangles (a) inlet and (b) outlet.

Solution:

Since this is an impulse turbine, assume coefficient of velocity = 0.98
Therefore the absolute velocity at inlet is

$$C_1 = 0.98\sqrt{2gH} = 0.98\sqrt{(2)(9.81)(35)} = 25.68 \text{ m/s}$$

The velocity of whirl at inlet

$$C_{\mathrm{w}1} = C_1 \cos \alpha_1 = 25.68 \cos 30° = 22.24 \text{ m/s}$$

Since $U_1 = U_2 = U$
Using outlet velocity triangle

$$C_2 = U_2 \tan \beta_2 = U \tan \beta_2 = U \tan 22°$$

Hydraulic efficiency of turbine (neglecting losses)

$$\eta_{\mathrm{h}} = \frac{C_{\mathrm{w}1} U_1}{gH} = \frac{H - C_2^2/2g}{H}$$

$$\frac{22.24U}{g} = H - \frac{(U \tan 22°)^2}{2g}$$

or

$$\frac{22.24U}{g} + \frac{(U \tan 22)^2}{2g} = H$$

or

$$22.24U + 0.082U^2 - 9.81H = 0$$

or

$$0.082U^2 + 22.24U - 9.81H = 0$$

or

$$U = \frac{-22.24 \pm \sqrt{(22.24)^2 + (4)(0.082)(9.81)(35)}}{(2)(0.082)}$$

As U is positive,

$$U = \frac{-22.24 + \sqrt{494.62 + 112.62}}{0.164}$$

$$= \frac{-22.24 + 24.64}{0.164} = 14.63 \text{ m/s}$$

Now using relation

$$U = \frac{\pi D N}{60}$$

or

$$N = \frac{60U}{\pi D} = \frac{(60)(14.63)}{(\pi)(1.5)} = 186\,\text{rpm}$$

Hydraulic efficiency

$$\eta_{\text{h}} = \frac{C_{\text{w1}}U}{gH} = \frac{(22.24)(14.63)}{(9.81)(35)} = 0.948 \text{ or } 94.8\%$$

Illustrative Example 3.11: A Kaplan runner develops 9000 kW under a head of 5.5 m. Assume a speed ratio of 2.08, flow ratio 0.68, and mechanical efficiency 85%. The hub diameter is 1/3 the diameter of runner. Find the diameter of the runner, and its speed and specific speed.

Solution:

$$U_1 = 2.08\sqrt{2gH} = 2.08\sqrt{(2)(9.81)(5.5)} = 21.61\text{ m/s}$$

$$C_{\text{r1}} = 0.68\sqrt{2gH} = 0.68\sqrt{(2)(9.81)(5.5)} = 7.06\text{ m/s}$$

Now power is given by

$$9000 = (9.81)(5.5)(0.85)Q$$

Therefore,

$$Q = 196.24\text{ m}^3/\text{s}$$

If D is the runner diameter and, d, the hub diameter

$$Q = \frac{\pi}{4}(D^2 - d^2)C_{\text{r1}}$$

or

$$\frac{\pi}{4}\left(D^2 - \frac{1}{9}D^2\right)7.06 = 196.24$$

Solving

$$D = \sqrt{\frac{(196.24)(4)(9)}{(\pi)(7.06)(8)}} = 6.31\text{ m}$$

$$N_{\text{S}} = \frac{N\sqrt{P}}{H^{5/4}} = \frac{65\sqrt{9000}}{5.5^{5/4}} = 732\text{ rpm}$$

Design Example 3.12: A propeller turbine develops 12,000 hp, and rotates at 145 rpm under a head of 20 m. The outer and hub diameters are 4 m and 1.75 m, respectively. Calculate the inlet and outlet blade angles measured at mean radius if overall and hydraulic efficiencies are 85% and 93%, respectively.

Solution:

$$\text{Mean diameter} = \frac{4 + 1.75}{2} = 2.875\,\text{m}$$

$$U_1 = \frac{\pi DN}{60} = \frac{(\pi)(2.875)(145)}{60} = 21.84\,\text{m/s}$$

Using hydraulic efficiency

$$\eta_h = \frac{C_{w1}U_1}{gH} = \frac{(C_{w1})(21.84)}{(9.81)(20)} = 0.93C_{w1}$$

or

$$C_{w1} = 8.35\,\text{m/s}$$

$$\text{Power} = (12,000)(0.746) = 8952\,\text{kW}$$

$$\text{Power} = \rho g Q H \eta_o$$

or

$$8952 = 9.81 \times Q \times 20 \times 0.85$$

Therefore, $Q = \dfrac{8952}{(9.81)(20)(0.85)} = 53.68\,\text{m}^3/\text{s}$

Discharge, $Q = 53.68 = \frac{\pi}{4}(4^2 - 1.75^2)C_{r1}$

$$\therefore\ C_{r1} = 5.28\,\text{m/s}$$

$$\tan\beta_1 = \frac{C_{r1}}{U_1 - C_{w1}} = \frac{5.28}{21.84 - 8.35} = \frac{5.28}{13.49} = 0.3914$$

$$\beta_1 = 21.38°$$

and

$$\tan\beta_2 = \frac{C_{r2}}{U_2} = \frac{5.28}{21.84} = 0.2418$$

$$\beta_2 = 13.59°$$

Illustrative Example 3.13: An inward flow reaction turbine wheel has outer and inner diameter are 1.4 m and 0.7 m respectively. The wheel has radial vanes and discharge is radial at outlet and the water enters the vanes at an angle of 12°. Assuming velocity of flow to be constant, and equal to 2.8 m/s, find

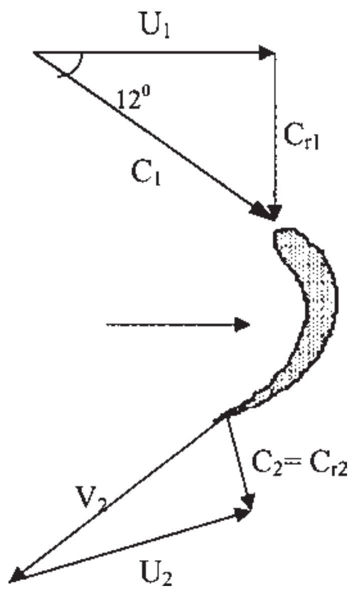

Figure 3.21 Velocity triangles at inlet and outlet for Example 3.13.

1. The speed of the wheel, and
2. The vane angle at outlet.

Solution:
Outer diameter, $D_2 = 1.4\,\text{m}$
Inner diameter, $D_1 = 0.7\,\text{m}$
Angle at which the water enters the vanes, $\alpha_1 = 12°$
Velocity of flow at inlet,

$$C_{r1} = C_{r2} = 2.8\,\text{m/s}$$

As the vanes are radial at inlet and outlet end, the velocity of whirl at inlet and outlet will be zero, as shown in Fig. 3.21.
Tangential velocity of wheel at inlet,

$$U_1 = \frac{C_{r1}}{\tan 12°} = \frac{2.8}{0.213} = 13.15\,\text{m/s}$$

Also, $U_1 = \frac{\pi D_2 N}{60}$ or

$$N = \frac{60U_1}{\pi D_2} = \frac{(60)(13.15)}{(\pi)(1.4)} = 179\,\text{rpm}$$

Let β_2 is the vane angle at outlet

$$U_2 = \frac{\pi D_1 N}{60} = \frac{(\pi)(0.7)(179)}{60} = 6.56\,\text{m/s}$$

From Outlet triangle,

$$\tan\beta_2 = \frac{C_{r2}}{U_2} = \frac{2.8}{6.56} = 0.4268 \;\text{ i.e. }\; \beta_2 = 23.11°$$

Illustrative Example 3.14: Consider an inward flow reaction turbine in which water is supplied at the rate of 500 L/s with a velocity of flow of 1.5 m/s. The velocity periphery at inlet is 20 m/s and velocity of whirl at inlet is 15 m/s. Assuming radial discharge, and velocity of flow to be constant, find

1. Vane angle at inlet, and
2. Head of water on the wheel.

Solution:

Discharge, $Q = 500\,\text{L/s} = 0.5\,\text{m}^3/\text{s}$

Velocity of flow at inlet, $C_{r1} = 1.5\,\text{m/s}$

Velocity of periphery at inlet, $U_1 = 20\,\text{m/s}$

Velocity of whirl at inlet, $C_{w1} = 15\,\text{m/s}$

As the velocity of flow is constant, $C_{r1} = C_{r2} = 1.5\,\text{m/s}$

Let β_1 = vane angle at inlet

From inlet velocity triangle

$$\tan(180 - \beta_1) = \frac{C_{r1}}{U_1 - C_{w1}} = \frac{1.5}{20 - 15} = 0.3$$

$$\therefore\; (180 - \beta_1) = 16°41'$$

or

$$\beta_1 = 180° - 16°41' = 163°19'$$

Since the discharge is radial at outlet, ad so the velocity of whirl at outlet is zero

Therefore,

$$\frac{C_{w1}U_1}{g} = H - \frac{C_1^2}{2g} = H - \frac{C_{r1}^2}{2g}$$

or

$$\frac{(15)(20)}{9.81} = H - \frac{1.5^2}{(2)(9.81)}$$

$$\therefore\ H = 30.58 - 0.1147 = 30.47\,\text{m}$$

Design Example 3.15: Inner and outer diameters of an outward flow reaction turbine wheel are 1 m and 2 m respectively. The water enters the vane at angle of 20° and leaves the vane radially. Assuming the velocity of flow remains constant at 12 m/s and wheel rotates at 290 rpm, find the vane angles at inlet and outlet.

Solution:

Inner diameter of wheel, $D_1 = 1\,\text{m}$

Outer diameter of wheel, $D_2 = 2\,\text{m}$

$\alpha_1 = 20°$

Velocity of flow is constant

That is, $C_{r1} = C_{r2} = 12\,\text{m/s}$

Speed of wheel, $N = 290\,\text{rpm}$

Vane angle at inlet $= \beta_1$

U_1 is the velocity of periphery at inlet.

Therefore, $U_1 = \frac{\pi D_1 N}{60} = \frac{(\pi)(1)(290)}{60} = 15.19\,\text{m/s}$

From inlet triangle, velocity of whirl is given by

$$C_{w1} = \frac{12}{\tan 20} = \frac{12}{0.364} = 32.97\,\text{m/s}$$

Hence, $\tan\beta_1 = \frac{C_{r1}}{C_{w1} - U_1} = \frac{12}{32.97 - 15.19} = \frac{12}{17.78} = 0.675$

i.e. $\beta_1 = 34°$

Let $\beta_2 =$ vane angle at outlet

$U_2 =$ velocity of periphery at outlet

Therefore $U_2 = \frac{\pi D_2 N}{60} = \frac{(\pi)(2)(290)}{60} = 30.38\,\text{m/s}$

From the outlet triangle

$$\tan\beta_2 = \frac{C_{r2}}{U_2} = \frac{12}{30.38} = 0.395$$

i.e.,

$$\beta_2 = 21°33'$$

Illustrative Example 3.16: An inward flow turbine is supplied with 245 L of water per second and works under a total head of 30 m. The velocity of wheel periphery at inlet is 16 m/s. The outlet pipe of the turbine is 28 cm in diameter. The radial velocity is constant. Neglecting friction, calculate

1. The vane angle at inlet
2. The guide blade angle
3. Power.

Solution:
If D_1 is the diameter of pipe, then discharge is

$$Q = \frac{\pi}{4} D_1^2 C_2$$

or

$$C_2 = \frac{(4)(0.245)}{(\pi)(0.28)^2} = 3.98 \text{ m/s}$$

But $C_2 = C_{r1} = C_{r2}$
Neglecting losses, we have

$$\frac{C_{w1} U_1}{gH} = \frac{H - C_2^2/2g}{H}$$

or

$$C_{W1} U_1 = gH - C_2^2/2$$

$$= [(9.81)(30)] - \frac{(3.98)^2}{2} = 294.3 - 7.92 = 286.38$$

Power developed

$$P = (286.38)(0.245) \text{ kW} = 70.16 \text{ kW}$$

and $C_{w1} = \dfrac{286.38}{16} = 17.9$ m/s

$$\tan \alpha_1 = \frac{3.98}{17.9} = 0.222$$

i.e. $\alpha_1 = 12°31'$

$$\tan \beta_1 = \frac{C_{r1}}{C_{w1} - U_1} = \frac{3.98}{17.9 - 16} = \frac{3.98}{1.9} = 2.095$$

i.e. $\beta_1 = 64.43$ or $\beta_1 = 64°25'$

Design Example 3.17: A reaction turbine is to be selected from the following data:

Discharge $= 7.8\,\mathrm{m^3/s}$

Shaft power $= 12{,}400\,\mathrm{kW}$

Pressure head in scroll casing

at the entrance to turbine $= 164\,\mathrm{m}$ of water

Elevation of turbine casing above tail water level $= 5.4\,\mathrm{m}$

Diameter of turbine casing $= 1\,\mathrm{m}$

Velocity in tail race $= 1.6\,\mathrm{m/s}$

Calculate the effective head on the turbine and the overall efficiency of the unit.

Solution:

Velocity in casing at inlet to turbine

$$C_c = \frac{\text{Discharge}}{\text{Cross} - \text{ sectional area of casing}}$$

$$= \frac{7.8}{(\pi/4)(1)^2} = 9.93\,\mathrm{m/s}$$

The net head on turbine

$$= \text{Pressure head} + \text{Head due to turbine position} + \frac{C_c^2 - C_1^2}{2g}$$

$$= 164 + 5.4 + \frac{(9.93)^2 - (1.6)^2}{2g}$$

$$= 164 + 5.4 + \frac{98.6 - 2.56}{19.62} = 174.3\,\mathrm{m} \text{ of water}$$

Waterpower supplied to turbine $= QgH$ kW

$$= (7.8)(9.81)(174.3) = 13{,}337\,\mathrm{kW}$$

Hence overall efficiency,

$$\eta_o = \frac{\text{Shaft Power}}{\text{Water Power}} = \frac{12{,}400}{13{,}337} = 0.93 \text{ or } 93\%$$

Design Example 3.18: A Francis turbine wheel rotates at 1250 rpm and net head across the turbine is 125 m. The volume flow rate is $0.45\,\mathrm{m^3/s}$, radius of the runner is 0.5 m. The height of the runner vanes at inlet is 0.035 m. and the angle of the inlet guide vanes is set at 70° from the radial direction. Assume that the absolute flow velocity is radial at exit, find the torque and power exerted by the water. Also calculate the hydraulic efficiency.

Solution:
For torque, using angular momentum equation

$$T = m(C_{W2}r_2 - C_{W1}r_1)$$

As the flow is radial at outlet, $C_{w2} = 0$ and therefore

$$\begin{aligned} T &= -mC_{W1}r_1 \\ &= -\rho QC_{W1}r_1 \\ &= -(10^3)(0.45)(0.5C_{W1}) \\ &= -225C_{W1}\text{Nm} \end{aligned}$$

If h_1 is the inlet runner height, then inlet area, A, is

$$\begin{aligned} A &= 2\pi r_1 h_1 \\ &= (2)(\pi)(0.5)(0.035) = 0.11\text{m}^2 \end{aligned}$$

$$C_{r1} = Q/A = \frac{0.45}{0.11} = 4.1\text{ m/s}$$

From velocity triangle, velocity of whirl

$$C_{w1} = C_{r1}\tan 70° = (4.1)(2.75) = 11.26\text{m/s}$$

Substituting C_{w1}, torque is given by

$$T = -(225)(11.26) = -2534\text{ Nm}$$

Negative sign indicates that torque is exerted on the fluid. The torque exerted by the fluid is +2534 Nm
Power exerted

$$\begin{aligned} P &= T\omega \\ &= \frac{(2534)(2)(\pi)(1250)}{(60)(1000)} \\ &= 331.83\text{ kW} \end{aligned}$$

Hydraulic efficiency is given by

$$\eta_h = \frac{\text{Power exerted}}{\text{Power available}}$$
$$= \frac{(331.83)(10^3)}{\rho g Q H}$$
$$= \frac{331.83 \times 10^3}{(10^3)(9.81)(0.45)(125)}$$
$$= 0.6013 = 60.13\%$$

Design Example 3.19: An inward radial flow turbine develops 130 kW under a head of 5 m. The flow velocity is 4 m/s and the runner tangential velocity at inlet is 9.6 m/s. The runner rotates at 230 rpm while hydraulic losses accounting for 20% of the energy available. Calculate the inlet guide vane exit angle, the inlet angle to the runner vane, the runner diameter at the inlet, and the height of the runner at inlet. Assume radial discharge, and overall efficiency equal to 72%.

Solution:
Hydraulic efficiency is

$$\eta_h = \frac{\text{Power deleloped}}{\text{Power available}}$$
$$= \frac{m(C_{w1}U_1 - C_{w2}U)}{\rho g Q H}$$

Since flow is radial at outlet, then $C_{w2} = 0$ and $m = \rho Q$, therefore

$$\eta_h = \frac{C_{w1}U_1}{gH}$$

$$0.80 = \frac{(C_{w1})(9.6)}{(9.81)(5)}$$

$$C_{w1} = \frac{(0.80)(9.81)(5)}{9.6} = 4.09\ \text{m/s}$$

Radial velocity $C_{r1} = 4$ m/s

$$\tan\alpha_1 = C_{r1}/C_{W1} \text{ (from velocity triangle)}$$
$$= \frac{4}{4.09} = 0.978$$

i.e., inlet guide vane angle $\alpha_1 = 44°21'$

$$\tan \beta_1 = \frac{C_{r1}}{(C_{w1} - U_1)}$$

$$= \frac{4}{(4.09 - 9.6)} = \frac{4}{-5.51} = -0.726$$

i.e., $\beta_1 = -35.98°$ or $180° - 35.98 = 144.02°$

Runner speed is

$$U_1 = \frac{\pi D_1 N}{60}$$

or

$$D_1 = \frac{60U_1}{\pi N} = \frac{(60)(9.6)}{(\pi)(230)}$$

$$D_1 = 0.797\,\text{m}$$

Overall efficiency

$$\eta_O = \frac{\text{Power output}}{\text{Power available}}$$

or

$$\rho g Q H = \frac{(130)(10^3)}{0.72}$$

or

$$Q = \frac{(130)(10^3)}{(0.72)(10^3)(9.81)(5)} = 3.68\,\text{m}^3/\text{s}$$

But

$$Q = \pi D_1 h_1 C_{r1} (\text{where } h_1 \text{ is the height of runner})$$

Therefore,

$$h_1 = \frac{3.68}{(\pi)(0.797)(4)} = 0.367\,\text{m}$$

Illustrative Example 3.20: The blade tip and hub diameters of an axial hydraulic turbine are 4.50 m and 2 m respectively. The turbine has a net head of 22 m across it and develops 22 MW at a speed of 150 rpm. If the hydraulic efficiency is 92% and the overall efficiency 84%, calculate the inlet and outlet blade angles at the mean radius assuming axial flow at outlet.

Solution:
Mean diameter, D_m, is given by

$$D_m = \frac{D_h + D_t}{2} = \frac{2 + 4.50}{2} = 3.25\,\text{m}$$

Overall efficiency, η_o, is given by

$$\eta_o = \frac{\text{Power develpoed}}{\text{Power available}}$$

$$\therefore\ \text{Power available} = \frac{22}{0.84} = 26.2\,\text{MW}$$

Also, available power $= \rho g Q H$

$$(26.2)(10^6) = (10^3)(9.81)(22)Q$$

Hence flow rate, Q, is given by

$$Q = \frac{(26.2)(10^6)}{(10^3)(9.81)(22)} = 121.4\,\text{m}^3/\text{s}$$

Now rotor speed at mean diameter

$$U_m = \frac{\pi D_m N}{60} = \frac{(\pi)(3.25)(150)}{60} = 25.54\,\text{m/s}$$

$$\begin{aligned}\text{Power given to runner} &= \text{Power available} \times \eta_h \\ &= 26.2 \times 10^6 \times 0.92 \\ &= 24.104\,\text{MW}\end{aligned}$$

Theoretical power given to runner can be found by using

$$P = \rho Q U_m C_{w1} (C_{w2} = 0)$$

$$(24.104)(10^6) = (10^3)(121.4)(25.54)(C_{w1})$$

$$\therefore\ C_{w1} = \frac{(24.104)(10^6)}{(10^3)(121.4)(25.54)} = 7.77\,\text{m/s}$$

Axial velocity is given by

$$C_r = \frac{Q \times 4}{\pi(D_t^2 - D_h^2)} = \frac{(121.4)(4)}{\pi(4.50^2 - 2^2)} = 9.51\,\text{m/s}$$

Using velocity triangle

$$\tan(180 - \beta_1) = \frac{C_r}{U_m - C_{w1}} = \frac{9.51}{25.54 - 7.77}$$

Inlet angle,

$$\beta_1 = 151.85^\circ$$

At outlet

$$\tan \beta_2 = \frac{C_r}{V_{cw_2}}$$

But V_{cw2} equals to U_m since C_{w2} is zero. Hence

$$\tan \beta_2 = \frac{9.51}{25.54} = 0.3724$$

that is,

$$\beta_2 = 20.43^\circ$$

Design Example 3.21: The following design data apply to an inward flow radial turbine:

Overall efficiency	75%
Net head across the turbine	6 m
Power output	128 kW
The runner tangential velocity	10.6 m/s
Flow velocity	4 m/s
Runner rotational speed	235 rpm
Hydraulic losses	18% of energy available

Calculate the inlet guide vane angle, the inlet angle of the runner vane, the runner diameter at inlet, and height of the runner at inlet. Assume that the discharge is radial.

Solution:

Hydraulic efficiency, η_h, is given by

$$\eta_h = \frac{\text{Power given to runner}}{\text{Water Power available}}$$

$$= \frac{m(U_1 C_{w1} - U_2 C_{w2})}{\rho g Q H}$$

Since flow is radial at exit, $C_{w2} = 0$ and $m = \rho Q$. Therefore

$$\eta_h = \frac{U_1 C_{w1}}{gH}$$

$$0.82 = \frac{(10.6)(C_{w1})}{(9.81)(6)} \quad \text{or} \quad C_{w1} = 4.6\,\text{m/s}$$

Now

$$\tan \alpha_1 = C_{r1}/C_{w1} = \frac{4}{4.6} = 0.8695$$

that is, $\alpha_1 = 41°$

From Figs. 3.22 and 3.23

$$\tan (180 - \beta_1) = \frac{C_{r1}}{U_1 - C_{w1}} = \frac{4}{10.6 - 4.6} = 0.667$$

that is, $\beta_1 = 33.69°$

Hence blade angle, β_1, is given by

$$180° - 33.69° = 146.31°$$

Runner speed at inlet

$$U_1 = \frac{\pi D_1 N}{60}$$

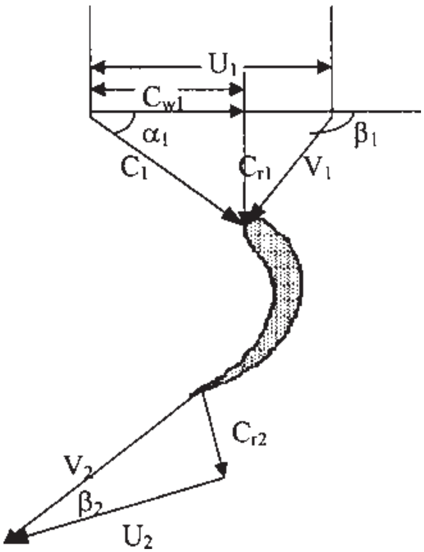

Figure 3.22 Velocity triangles for Example 3.14.

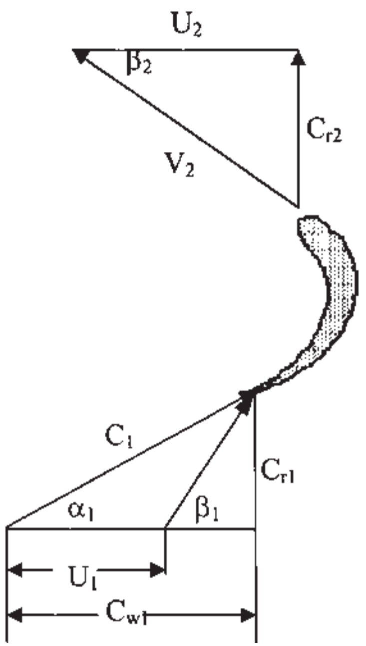

Figure 3.23 Velocity triangles at inlet and outlet for Example 3.15.

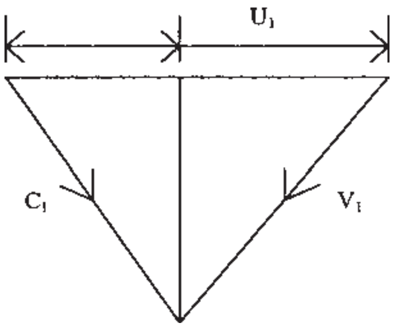

Figure 3.24 Inlet velocity triangle.

or

$$D_1 = \frac{U_1(60)}{\pi N} = \frac{(10.6)(60)}{(\pi)(235)} = 0.86\,\text{m}$$

Overall efficiency

$$\eta_O = \frac{\text{Power output}}{\text{Power available}}$$

$$\rho g Q H = \frac{(128)(10^3)}{0.75}$$

From which flow rate

$$Q = \frac{(128)(10^3)}{(0.75)(10^3)(9.81)(6)} = 2.9\,\text{m}^3/\text{s}$$

Also,

$$Q = \pi D_1 h C_{r1}$$

where h_1 is the height of runner
Therefore,

$$h_1 = \frac{2.9}{(\pi)(0.86)(4)} = 0.268\,\text{m}$$

Design Example 3.22: A Kaplan turbine develops 10,000 kW under an effective head 8 m. The overall efficiency is 0.86, the speed ratio 2.0, and flow ratio 0.60. The hub diameter of the wheel is 0.35 times the outside diameter of the wheel. Find the diameter and speed of the turbine.

Solution:
Head, H = 8 m, Power, P = 10,000 kW
Overall efficiency, $\eta_o = 0.86$
Speed ratio

$$2 = \frac{U_1}{(2gH)^{1/2}}, \quad \text{or } U_l = \sqrt{2 \times 9.81 \times 8} = 25.06\,\text{m/s}$$

Flow ratio

$$\frac{C_{r1}}{(2gH)^{1/2}} = 0.60 \quad \text{or } C_{r1} = 0.60\sqrt{2 \times 9.81 \times 8} = 7.52\,\text{m/s}$$

Hub diameter, $D_1 = 0.35\ D_2$

Overall efficiency,

$$\eta_O = \frac{P}{\rho g Q H}$$

Or

$$0.86 = \frac{10000}{1000 \times 9.81 \times Q \times 8}$$

$$\therefore \ \mathrm{Q} = 148.16\,\mathrm{m}^3/\mathrm{s}$$

Now using the relation

$$\mathrm{Q} = C_{r1} \times \frac{\pi}{4}\left[D_1^2 - D_2^2\right]$$

Or

$$148.16 = 7.52 \times \frac{\pi}{4}\left[D_1^2 - \left(0.35 D_1^2\right)\right]$$

$$\therefore \ \mathrm{D}_1 = 5.35\,\mathrm{m}$$

The peripheral velocity of the turbine at inlet

$$25.06 = \frac{\pi D_1 N}{60} = \frac{\pi \times 5.35 \times N}{60}$$

$$\therefore \ \mathrm{N} = \frac{60 \times 25.06}{\pi \times 5.35} = 89\,\mathrm{rpm}$$

Design Example 3.23: An inward flow reaction turbine, having an inner and outer diameter of 0.45 m and 0.90 m, respectively. The vanes are radial at inlet and the discharge is radial at outlet and the water enters the vanes at an angle of 12°. Assuming the velocity of flow as constant and equal to 2.8 m/s, find the speed of the wheel and the vane angle at outlet.

Solution:
Inner Diameter, $\mathrm{D}_2 = 0.45\,\mathrm{m}$
Outer Diameter, $\mathrm{D}_1 = 0.9\,\mathrm{m}$

$$\alpha_2 = 90°\text{(radial discharge)}$$

$$\alpha_1 = 12°, \mathrm{C}_{r1} = \mathrm{C}_{r2} = 2.8\,\mathrm{m/s}$$

From velocity triangle at inlet (see Fig. 3.11), The peripheral velocity of the wheel at inlet

$$U_1 = \frac{C_{r1}}{\tan \alpha_1} = \frac{2.8}{\tan 12^\circ} = 13.173 \text{ m/s}$$

Now,

$$U_1 = \frac{\pi D_1 N}{60}$$

or

$$N = \frac{60 U_1}{\pi D_1} = \frac{60 \times 13.173}{\pi \times 0.9} = 279 \text{ rpm}$$

Considering velocity triangle at outlet peripheral velocity at outlet

$$U_2 = \frac{\pi D_2 N}{60} = \frac{\pi \times 0.45 \times 279}{60} = 6.58 \text{ m/s}$$

$$\tan \beta_2 = \frac{C_{r2}}{U_2} = \frac{2.8}{6.58} = 0.426$$

$$\therefore \ \beta_2 = 23.05^\circ$$

Design Example 3.24: An inward flow reaction turbine develops 70 kW at 370 rpm. The inner and outer diameters of the wheel are 40 and 80 cm, respectively. The velocity of the water at exit is 2.8 m/s. Assuming that the discharge is radial and that the width of the wheel is constant, find the actual and theoretical hydraulic efficiencies of the turbine and the inlet angles of the guide and wheel vanes. Turbine discharges 545 L/s under a head of 14 m.

Solution:

$Q = 545 \text{ L/s} = 0.545 \text{ m}^3/\text{s}$

$D_1 = 80 \text{ cm}, D_2 = 40 \text{ cm}$

$H = 14 \text{ m}, \alpha_2 = 90^\circ$ (radial discharge)

$\beta_1 = \beta_2$

Peripheral velocity of the wheel at inlet

$$U_1 = \frac{\pi D_1 N}{60} = \frac{\pi \times 0.80 \times 370}{60} = 15.5 \text{ m/s}$$

Velocity of flow at the exit, $C_{r2} = 2.8$ m/s

As $\alpha_2 = 90^\circ$, $C_{r2} = C_2$

Work done/s by the turbine per kg of water $= \frac{C_w \times U_1}{g}$

But this is equal to the head utilized by the turbine, i.e.

$$\frac{C_{w1}U_1}{g} = H - \frac{C_2}{2g}$$

(Assuming there is no loss of pressure at outlet) or

$$\frac{C_{w1} \times 15.5}{9.81} = 14 - \frac{(2.8)^2}{2 \times 9.81} = 13.6\,\text{m}$$

or

$$C_{w1} = \frac{13.6 \times 9.81}{15.5} = 8.6\,\text{m/s}$$

Work done per second by turbine

$$= \frac{\rho Q}{g} C_{w1} U_1$$

$$= \frac{1000 \times 0.545 \times 8.6 \times 15.5}{1000}$$

$$= 72.65\text{kW}$$

$$\text{Available power or water power} = \frac{\rho g Q H}{1000}$$

$$= 74.85$$

Actual available power = 70 kW

$$\text{Overall turbine efficiency is } \eta_t = \frac{70}{74.85} \times 100$$

$$\eta_t = 93.52\%$$

This is the actual hydraulic efficiency as required in the problem.
Hydraulic Efficiency is

$$\eta_h = \frac{72.65}{75.85} \times 100 = 97.06\%$$

This is the theoretical efficiency

$$Q = \pi D_1 b_1 C_{r1} = \pi D_2 b_2 C_{r2}$$

(Neglecting blade thickness)

$$C_{r1} = C_{r2}\frac{D_2}{D_1} = 2.8 \times \frac{40}{20} = 1.4\,\text{m/s}$$

Drawing inlet velocity triangle

$$\tan \beta_1 = \frac{C_{r1}}{U_1 - C_{w1}} = \frac{1.4}{15.5 - 8.6} = \frac{1.4}{6.9} = 0.203$$

i.e., $\beta_1 = 11.47°$

$$C_1 = \sqrt{C_{w1} + C_{r1}} = \left(8.6^2 + 1.4^2\right)^{0.5} = 8.64\ \text{m/s}$$

and

$$\cos\alpha_1 = \frac{C_{w1}}{C_1} = \frac{8.6}{8.64} = 0.995$$

i.e., $\alpha_1 = 5.5°$

Design Example 3.25: An inward flow Francis turbine, having an overall efficiency of 86%, hydraulic efficiency of 90%, and radial velocity of flow at inlet $0.28\sqrt{2gH}$. The turbine is required to develop 5000 kW when operating under a net head of 30 m, specific speed is 270, assume guide vane angle 30°, find

1. rpm of the wheel,
2. the diameter and the width of the runner at inlet, and
3. the theoretical inlet angle of the runner vanes.

Solution:

Power, $P = 5000\ \text{kW}, \alpha_1 = 30°, H = 30\ \text{m}, C_{r1} = 0.28\sqrt{2gH}$, $N_s = 270$, $\eta_h = 0.90$, $\eta_o = 0.86$

1. Specific speed of the turbine is

$$N_S = \frac{N\sqrt{P}}{H^{5/4}}$$

or

$$N = \frac{N_S H^{5/4}}{\sqrt{P}} = \frac{270 \times (30)^{1.25}}{\sqrt{5000}} = \frac{18957}{71} = 267\ \text{rpm}$$

2. Velocity of Flow:

$$C_{r1} = 0.28\sqrt{2 \times 9.81 \times 30} = 6.79\text{m/s}$$

From inlet velocity triangle

$$C_{r1} = C_1 sin\,\alpha_1$$

or

$$6.79 = C_1 \sin 30°$$

or

$$C_1 = \frac{6.79}{0.5} = 13.58\ \text{m/s}$$

$$C_{w1} = C_1 \cos 30° = 13.58 \times 0.866 = 11.76\ \text{m/s}$$

Work done per (sec) (kg) of water

$$= \frac{C_{w1} \times U_1}{g} = \eta_h \times H$$

$$= 0.9 \times 30$$

$$= 27 \text{ mkg/s}$$

Peripheral Velocity,

$$U_1 = \frac{27 \times 9.81}{11.76} = 22.5 \text{ m/s}$$

But $U_1 = \dfrac{\pi D_1 N}{60}$
or

$$D_1 = \frac{60 U_1}{\pi N} = \frac{60 \times 22.5}{\pi \times 267} = 1.61 \text{ m}$$

Power, $P = \rho g Q H \eta_o$
or
$5000 = 1000 \times 9.81 \times Q \times 30 \times 0.86$
or
$Q = 19.8 \text{ m}^3/\text{s}$
Also $Q = k\pi D_1 b_1 C_{r1}$ (where k is the blade thickness coefficient and b_1 is the breath of the wheel at inlet) or

$$b_1 = \frac{Q}{k\pi D_1 C_{r1}} = \frac{19.8}{0.95 \times \pi \times 1.61 \times 6.79} = 0.61 \text{ m}$$

3. From inlet velocity triangle

$$\tan \beta_1 = \frac{C_{r1}}{U_1 - C_{w1}} = \frac{6.79}{22.5 - 11.76} = \frac{6.79}{10.74} = 0.632$$

i.e. $\beta_1 = 32.30°$

Design Example 3.26: A 35 MW generator is to operate by a double overhung Pelton wheel. The effective head is 350 m at the base of the nozzle. Find the size of jet, mean diameter of the runner and specific speed of wheel. Assume Pelton wheel efficiency 84%, velocity coefficient of nozzle 0.96, jet ratio 12, and speed ratio 0.45.

Solution:
In this case, the generator is fed by two Pelton turbines.

Power developed by each turbine,

$$P_{\mathrm{T}} = \frac{35,000}{2} = 17,500\,\mathrm{kW}$$

Using Pelton wheel efficiency in order to find available power of each turbine

$$P = \frac{17,500}{0.84} = 20,833\,\mathrm{kW}$$

But, $P = \rho gQH$

$$Q = \frac{P}{\rho gH} = \frac{20833}{1000 \times 9.81 \times 350} = 6.07\,\mathrm{m^3/s}$$

Velocity of jet, $C_{\mathrm{j}} = C_{\mathrm{V}}\sqrt{2gH} = 0.96\sqrt{2 \times 9.81 \times 350}$

$$C_{\mathrm{j}} = 79.6\,\mathrm{m/s}$$

Area of jet, $A = \frac{Q}{C_j} = \frac{6.07}{79.6} = 0.0763\,\mathrm{m^2}$

$$\therefore \text{ Diameter of jet}, d = \left(\frac{4A}{\pi}\right)^{0.5} = \left(\frac{4 \times 0.0763}{\pi}\right)^{0.5} = 0.312\,\mathrm{m}$$

$$d = 31.2\,\mathrm{cm}$$

Diameter of wheel D = d × jet ratio = 0.312 × 12 = 3.744 m
Peripheral velocity of the wheel

$$U = \text{speed ratio}\sqrt{2gH}$$

$$= 0.45 \times \sqrt{2 \times 9.81 \times 350} = 37.29\,\mathrm{m/s}$$

But $U = \frac{\pi DN}{60}$ or

$$N = \frac{60U}{\pi D} = \frac{60 \times 37.29}{\pi \times 3.744} = 190\,\mathrm{rpm}$$

Specific speed,

$$N_{\mathrm{S}} = \frac{N\sqrt{P_T}}{H^{5/4}} = \frac{190\sqrt{17,500}}{(350)^{1.25}} = 16.6$$

PROBLEMS

3.1 A Pelton wheel produces 4600 hP under a head of 95 m, and with an overall efficiency of 84%. Find the diameter of the nozzle if the coefficient of velocity for the nozzle is 0.98.

(0.36 m)

3.2 Pelton wheel develops 13,500 kW under a head of 500 m. The wheel rotates at 430 rpm. Find the size of the jet and the specific speed. Assume 85% efficiency.

(0.21 m, 21)

3.3 A Pelton wheel develops 2800 bhP under a head of 300 m at 84% efficiency. The ratio of peripheral velocity of wheel to jet velocity is 0.45 and specific speed is 17. Assume any necessary data and find the jet diameter.

(140 mm)

3.4 A Pelton wheel of power station develops 30,500 hP under a head of 1750 m while running at 760 rpm. Calculate (1) the mean diameter of the runner, (2) the jet diameter, and (3) the diameter ratio.

(2.14 m, 0.104 m, 20.6)

3.5 Show that in an inward flow reaction turbine, when the velocity of flow is constant and wheel vane angle at entrance is 90°, the best peripheral velocity is

$$\sqrt{2gH}/\sqrt{2 + \tan^2\alpha}$$

where H is the head and α the angle of guide vane.

3.6 A Pelton wheel develops 740 kW under a head of 310 m. Find the jet diameter if its efficiency is 86% and

$$C_V = 0.98.$$

(0.069 m)

3.7 A reaction turbine runner diameter is 3.5 m at inlet and 2.5 m at outlet. The turbine discharge 102 m^3 per second of water under a head of 145 m. Its inlet vane angle is 120°. Assume radial discharge at 14 m/s, breadth of wheel constant and hydraulic efficiency of 88%, calculate the power developed and speed of machine.

(128 MW, 356 rpm)

3.8 Show that in a Pelton wheel, where the buckets deflect the water through an angle of (180° − α) degrees, the hydraulic efficiency of the wheel is given by

$$\eta_h = \frac{2U(C - U)(1 + \cos\alpha)}{C^2}$$

where C is the velocity of jet and U is mean blade velocity.

3.9 A Kaplan turbine produces 16000 kW under a head of 20 m, while running at 166 rpm. The diameter of the runner is 4.2 m while the hub diameter is 2 m, the discharge being 120 m^3/s. Calculate (1) the turbine efficiency,

(2) specific speed, (3) the speed ratio based on the tip diameter of the blade, and (4) the flow ratio.

(78%, 497, 1.84, 0.48)

3.10 Evolve a formula for the specific speed of a Pelton wheel in the following form

$$N_s = k \cdot \sqrt{\eta} \cdot \frac{d}{D}$$

where N_s = specific speed, η = overall efficiency, d = diameter of jet, D = diameter of bucket circle, and k = a constant.

NOTATION

C	jet velocity, absolute
C_v	nozzle velocity coefficient
C_w	velocity of whirl
D	wheel diameter
d	diameter of nozzle
E	energy transfer by bucket
H_r	head across the runner
h_f	frictional head loss
N_s	specific speed
P	water power available
P_c	casing and draft tube losses
P_h	hydraulic power loss
P_l	leakage loss
P_m	mechanical power loss
P_r	runner power loss
P_s	shaft power output
U	bucket speed
W	work done
α	angle of the blade tip at outlet
β	angle with relative velocity
η_i	nozzle efficiency
η_{trans}	transmission efficiency
κ	relative velocity ratio